

# How to Complement the Description of Physical Universe?

*T.F.Kamalov*

Physics Department

Moscow State Opened University

107996 Moscow, 22, P. Korchagin str., Russia

E-mail: ykamalov@rambler.ru

## Abstract

Which non-local hidden variables could complement the description of physical Universe? The model of extended Newtonian dynamics is presented.

Keywords: Model of Extended Newtonian Dynamics, MEND, non-local hidden variables.

PACS: 03.65.Ud

## 1 Introduction

Classical Newtonian mechanics is essentially the simplest way of mechanical system description with second-order differential equations, when higher order time derivatives of coordinates can be neglected. The extended model of Newtonian mechanics with higher time derivatives of coordinates is based on generalization of Newton's classical axiomatics onto arbitrary reference systems (both inertial and non-inertial ones) with body dynamics being described with higher order differential equations. Newton's Laws, constituting, from the mathematical viewpoint, the axiomatics of classical physics, actually postulate the assertion that the equations describing the dynamics of bodies in non-inertial systems are second-order differential equations. However, the actual time-space is almost without exception non-inertial, as it is almost without exception that there exist (at least weak) fields, waves, or forces perturbing an ideal inertial system. Non-inertial nature of the actual time-space is also supported by observations of the practical astronomy that expansion of the Universe occurs with an acceleration. In other words, actually any real reference system is a non-inertial one; and such physical reality can be described with a differential equation with time derivatives of coordinates of the order exceeding two, which play the role of additional variables. This is evidently beyond the scope of Newtonian axiomatics. Aristotle's physics considered velocity to be proportional to the applied force, hence the body dynamics was described by first derivative differential equation. Newtonian axiomatics postulates inertial reference systems, where a free body maintains the constant velocity of translational motion. The body dynamics is described with a second order differential equation, with acceleration being proportional to force [1]. This corresponds to the Lagrangian depending on coordinates and their first derivatives (velocities) of the body, and Euler-Lagrange equation resulting from the principle of the least action. This model of the physical reality describes macrocosm fairly good, but it fails to describe micro

particles. Both Newtonian axiomatics and the Second Law of Newton are invalid in microcosm. Only averaged values of observable physical quantities yield in the microcosm the approximate analog of the Second Law of Newton; this is the so-called Ehrenfest's theorem. The Ehrenfest's equation yields the averaged, rather than precise, relation between the second time derivative of coordinate and the force, while to describe the scatter of quantum observables the probability theory apparatus is required. As the Newtonian dynamics is restricted to the second order derivatives, while micro-objects must be described with equations with additional variables, tending Planck's constant to zero corresponds to neglecting these variables. Hence, offering the model of extended Newtonian dynamics, we consider classical and quantum theories with additional variables, describing the body dynamics with higher order differential equations. In our model the Lagrangian shall be considered depending not only on coordinates and their first time derivatives, but also on higher-order time derivatives of coordinates. Classical dynamics of test particle motion with higher-order time derivatives of coordinates was first described in 1850 by M. Ostrogradskii [2] and is known as Ostrogradskii's Canonical Formalism. Being a mathematician, M. Ostrogradskii considered coordinate systems rather than reference systems. This is just the case corresponding to a real reference system comprising both inertial and non-inertial reference systems. In a general case, the Lagrangian takes on the form

$$L = L(t, q, \dot{q}, \ddot{q}, \dots, \dot{q}^n). \quad (1)$$

## 2 Model of Extended Newtonian Dynamics

Let us consider in more detail this precise description of the dynamics of body motion, taking into account of real reference systems. To describe the extended dynamics of a body in an arbitrary coordinate system (corresponding to any reference system) let us introduce concepts of kinematic state and kinematic invariant of an arbitrary reference system.

**Definition:** Kinematic state of a body is set by  $n$ -th time derivative of coordinate. The kinematic state of the body is defined provided the  $n$ -th time derivative of body coordinate is zero, the  $(n - 1)$ -th time derivative of body coordinate being constant. In other words, we consider the kinematic state of the body defined if  $(n - 1)$ -th time derivative of body coordinate is finite. Let us note that a reference system performing harmonic oscillations with respect to an inertial reference system does not possess any definite kinematic state. Considering the dynamics of particles in any reference systems, we suggest the following two postulates.

**Postulate 1.** Kinematic state of a free body is invariable. This means that if the  $n$ -th time derivative of a free body coordinate is zero, the  $(n - 1)$ -th time derivative of body coordinate is constant. That is,

$$\frac{d^n q}{dt^n} = 0, \frac{d^{n-1} q}{dt^{n-1}} = const. \quad (2)$$

In the extended model of dynamics, conversion from a reference system to another one will be defined as:

$$q' = q_0 + \dot{q}t + \frac{1}{2!}\ddot{q}t^2 + \dots + \frac{1}{n!}\dot{q}^{(n)}t^n \quad (3)$$

$$t' = t. \quad (4)$$

**Postulate 2.** If the kinematic invariant of a reference system is  $n$ -th time derivative of body coordinate, then the body dynamics is described with the differential equation of the order  $2n$ :

$$\alpha_{2n}\dot{q}^{(2n)} + \dots + \alpha_0 q = F(t, q, \dot{q}, \ddot{q}, \dots, \dot{q}^{(n)}). \quad (5)$$

This means that the Lagrangian depends on  $n$ -th time derivative of coordinate, so variation when applying the least action principle will yield the order higher by a unity. Therefore, the dynamics of a free body in a reference system with  $n$ -th order derivative being invariant shall be described with a differential equation of the order  $2n$ . To consider dynamics of a body with an observer in an arbitrary coordinate system (which corresponds to the case of any reference system), we apply the least action principle, varying the action function for  $n$ -th order kinematic invariant, we obtain the equation of the order  $2n$ :

$$\delta S = \delta \int L(t, \dot{q}', q') dt = \int \sum_{n=0}^N (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial \dot{q}^{(n)}} \delta \dot{q}^{(n)} dt = 0. \quad (6)$$

Then the equation describing the dynamics of a body with  $n$ -invariant is a  $2n$ -order differential equation, and for the case of irreversible time arrow we shall retain only even components. Expanding into Taylor's series the function  $q = q(t)$  yields:

$$q = q_0 + \dot{q}t + \frac{1}{2!}\ddot{q}t^2 + \dots + \frac{1}{n!}\dot{q}^{(n)}t^n. \quad (7)$$

It is well known that the kinematic equation in inertial reference systems of Newtonian physics contains the second time derivative of coordinate, that is, acceleration:

$$q_{Newton} = q_0 + vt + \frac{1}{2}at^2. \quad (8)$$

Let us denote the additional terms with higher derivatives as

$$q_r = \frac{1}{3!}\dot{q}^{(3)}t^3 + \dots + \frac{1}{n!}\dot{q}^{(n)}t^n. \quad (9)$$

Then

$$q = q_{newton} + q_r. \quad (10)$$

In our case, the discrepancy between descriptions of the two models is the difference between the description of test particles in the model of extended

Newtonian dynamics with Lagrangian  $L(t, q, \dot{q}, \ddot{q}, \dots, \dot{q}^{(n)}, \dots)$  and Newtonian dynamics in inertial reference systems with the Lagrangian  $L(t, q, \dot{q})$ :

$$\int [L(t, q, \dot{q}, \ddot{q}, \dots, \dot{q}^{(n)}) - L(t, q, \dot{q})] dt = h, \quad (11)$$

$h$  being the discrepancy (error) between descriptions by the two models. Comparing this value with the uncertainty of measurement in inertial reference systems, expressed by the Heisenberg uncertainty relation, the equation (11) can be rewritten as

$$S(t, q, \dot{q}, \dots, \dot{q}^{(n)}) - S(t, q, \dot{q}) = h. \quad (12)$$

In the classical mechanics, in inertial reference systems, the Lagrangian depends only on the coordinates and their first time derivatives. In the extended models, in real reference systems, the Lagrangian depends not only on the coordinates and their first time derivatives, but also on their higher derivatives. Applying the least action principle [3], we obtain Euler-Lagrange equation for the extended Newtonian dynamics model:

$$\sum_{n=0}^N (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial \dot{q}^{(n)}} = 0, \quad (13)$$

or

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}} - \dots + (-1)^N \frac{d^N}{dt^N} \frac{\partial L}{\partial \dot{q}^{(N)}} = 0. \quad (14)$$

The Lagrangian will be expressed through quadratic functions of variables:

$$L = kq^2 - k_1 \dot{q}^2 + k_2 \ddot{q}^2 - \dots + (-1)^\alpha k_\alpha \dot{q}^{(\alpha)2} = \sum_{\alpha=0}^{\infty} (-1)^\alpha k_\alpha \dot{q}^{(\alpha)2}. \quad (15)$$

For our case, the action function will be:

$$S = q \frac{\partial L}{\partial q} - \dot{q} \frac{\partial L}{\partial \dot{q}} + \dots + (-1)^\alpha \dot{q}^{(\alpha)} \frac{\partial \dot{L}^{(\alpha)}}{\partial \dot{q}^{(\alpha)}} + \dots = \sum_{\alpha=0}^{\infty} (-1)^\alpha \dot{q}^{(\alpha)} \frac{d^\alpha}{dt^\alpha} \frac{\partial L}{\partial \dot{q}^{(\alpha)}}. \quad (16)$$

Or

$$S = 2kq^2 - 2k_1 \dot{q}^2 + 2k_2 \ddot{q}^2 - \dots + 2k_\alpha \dot{q}^{(\alpha)2} = 2 \sum_{\alpha=0}^{\infty} (-1)^\alpha k_\alpha \dot{q}^{(\alpha)2}. \quad (17)$$

In the space with curvature,

$$L = \sum_{\alpha=0}^{\infty} (-1)^\alpha g_{ik} \dot{q}^{(\alpha)i} \dot{q}^{(\alpha)k}. \quad (18)$$

Here, instead of Schwarzschild metric

$$ds^2 = (1 - \frac{q_g}{q}) c^2 dt^2 - (1 - \frac{q_g}{r}) dq^2 - q^2 d\theta - q^2 \sin^2 \theta d\phi^2, \quad (19)$$

we will use the metric

$$ds^2 = \exp(-\frac{q_g}{q})c^2 dt^2 - \exp(-\frac{q_g}{q})dq^2 - q^2 d\theta - q^2 \sin^2 \theta d\phi^2, \quad (20)$$

where

$$q_g = \frac{2GM}{c^2}, g_{00} = \exp(-\frac{q_g}{q}). \quad (21)$$

Introducing the notation

$$F = \frac{\partial L}{\partial q}, p = \frac{\partial L}{\partial \dot{q}} \quad (22)$$

$$F^2 = \frac{\partial L}{\partial \ddot{q}}, p^3 = \frac{\partial L}{\partial \dot{q}^{(3)}} \quad (23)$$

$$F^4 = \frac{\partial L}{\partial \dot{q}^{(4)}}, p^5 = \frac{\partial L}{\partial \dot{q}^{(5)}} \quad (24)$$

.....

$$F^{2n} = \frac{\partial L}{\partial \dot{q}^{(2n)}}, p^{2n+1} = \frac{\partial L}{\partial \dot{q}^{(2n+1)}}, \quad (25)$$

we obtain the description of inertial forces for the extended Newtonian dynamics model. The value of the resulting force accounting for inertial forces can be expressed through momentums and their derivatives, expressing the Second Law of Newton for the extended Newtonian dynamics model:

$$F - \frac{dp}{dt} + \frac{d^2}{dt^2}(F^2 - \frac{dp^3}{dt}) + \frac{d^4}{dt^4}(F^4 - \frac{dp^5}{dt}) + \dots \frac{d^n}{dt^n}(F^n - \frac{dp^{n+1}}{dt}) = 0. \quad (26)$$

Expanding the force into Taylor series, we obtain:

$$F(t) = F_0 + \dot{F}t + \frac{1}{2!}\ddot{F}t^2 + \dots \quad (27)$$

In other words, (26) can be written as

$$\sum_{n=0}^{\infty} \frac{d^{2n}}{dt^{2n}}(F^{2n} - \frac{d^{2n}p^{2n+1}}{dt^{2n}}) = 0. \quad (28)$$

The action function takes on the form

$$S = \sum_{n=0}^{\infty} (-1)^n \dot{q}^{(n)} p^{n+1} = \sum_{n=0}^N (-1)^n \dot{q}^{(n)} \frac{\partial L}{\partial \dot{q}^{(n+1)}}. \quad (29)$$

The Hamiltonian will be

$$H = \sum_{n=0}^{\infty} \dot{q}^{(n)} p^{n+1}. \quad (30)$$

For this case, energy can be expressed as

$$E = \alpha_0 q^2 + \alpha_1 \dot{q}^2 + \alpha_2 \ddot{q}^2 + \dots + \alpha_n \dot{q}^{(n)2} + \dots \quad (31)$$

Denoting the Appel's energy of acceleration [4] as  $Q$ ,  $\alpha_n$  being constant factors, we obtain for kinetic energy and potential energy, respectively,

$$E = V + W + Q \quad (32)$$

$$V = \alpha_0 q^2, \quad (33)$$

$$W = \alpha_1 \dot{q}^2 \quad (34)$$

$$Q = \alpha_2 \ddot{q}^2 + \dots + \alpha_n \dot{q}^{(n)2} + \dots \quad (35)$$

The Hamilton-Jacobi equation for the action function will take on the form

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + Q, \quad (36)$$

We will call  $Q$  the quantum potential. The first addend in (35) is the so-called Appel's energy of acceleration [4]. The coordinate derivative of the quantum potential yields the force:

$$F_Q = -\frac{\partial Q}{\partial q}. \quad (37)$$

The generalized action function will take on the form

$$S = qp + \dot{q}p^1 + \ddot{q}p^2 + \dots \quad (38)$$

The velocity is  $v = \frac{\partial S}{\partial t}$ , and  $\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial q} = 0$ , being the continuity equation of velocity vector. Now, denoting  $\frac{\partial S}{\partial \dot{q}^{(n)}} = p^{n-1}$ , we obtain the equation

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \frac{\partial S}{\partial q} \dot{q} + \frac{\partial S}{\partial \dot{q}} \ddot{q} + \dots + \frac{\partial S}{\partial \dot{q}^{(n)}} \dot{q}^{(n+1)}, \quad (39)$$

or

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \frac{\partial S}{\partial q} \frac{\nabla S}{2m} + p^1 \ddot{q} + \dots + p^n \dot{q}^{(n+1)}. \quad (40)$$

Then

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q. \quad (41)$$

Here, the additional potential  $Q$

$$Q = p^1 \ddot{q} + \dots + p^n \dot{q}^{(n+1)}. \quad (42)$$

Let us complement the equation (2.41) with the continuity equation [5]. In the first approximation,  $Q \approx \alpha_3 \frac{\nabla^2 S}{m^2}$  (here, the value of the constant is chosen  $\alpha_3 = \frac{i\hbar m}{2}$ ). Hence, in the first approximation we obtain for the function

$$\psi = e^{\frac{i}{\hbar} S}, \quad (43)$$

the Schroedinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi + V\psi. \quad (44)$$

In our case, for free classical particles, we obtain the oscillation equation:

$$\sum_{n=0}^N k_{2n} \dot{q}^{(2n)} = 0, \quad (45)$$

or

$$k_0 q^2 + k_2 \ddot{q}^2 + \dots + k_{2N} \dot{q}^{(2N)2} = 0. \quad (46)$$

The equation (2.46) will take on the form

$$\omega_0^2 q^2 + \omega_2 \ddot{q}^2 + \dots + \dot{q}^{(2N)2} = 0, \quad (47)$$

provided we introduce the notation

$$\omega_{2n} = k_{2n}/k_{2N}. \quad (48)$$

### 3 Conclusions

Our case corresponds to Lagrangian  $L(t, q, \dot{q}, \ddot{q}, \dots, \dot{q}^{(n)}, \dots)$ , depending on coordinates, velocities and higher time derivatives, which we call additional variables, extra addends, or hidden variables. In arbitrary reference systems (including non-inertial ones) additional variables (addends) appear in the form of higher time derivatives of coordinates, which complement both classical and quantum physics. We call these additional addends, or variables, constituting the higher time derivatives of coordinates, hidden variables or hidden parameters, complementing the description of particles. It should be noted that these hidden parameters can be used to complement the quantum description without violating von Neumann theorem, as this theorem is not applied for non-linear reference systems, while the extended Newtonian dynamics model assumes employing any reference systems, including non-linear ones. Comparing the generalized Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + Q, \quad (49)$$

$Q$  being the additional variables with higher derivatives, with the quantum Bohm's potential, one can conclude that neglecting higher-order time derivatives of coordinates brings about incompleteness of physical Universe description. The coordinate derivative of  $Q$  determines the quantum force. This means that complete description of physical Universe requires considering differential equations of the order exceeding second; uncertainty of the position of the particle under investigation shall be attributed to fluctuations of the reference body

and reference system associated with it. Hence, the differential equation describing this case shall be of the order exceeding second. In this case, uncertainty of a micro objects description is follow by incompleteness of the description of the physical Universe by Newtonian physics, that is, the lack of a complete description with additional variables in the form of higher time derivatives of coordinates. The contemporary physics presupposes employment of predominantly inertial reference systems; however, such a system is very hard to obtain, as there always exist external perturbative effects, for example, gravitational forces, fields, or waves. In this case, the relativity principle enables transfer from the gravitational forces or waves to inertial forces. For example, if we consider a spaceship with two observers in different cabins, one can see that this system is non-ideal, the inertial forces (or pseudo-forces) could constitute additional parameters here. In this case, superposition of the two distributions obtained by the observers could yield a non-zero correlation factor, though each of the two observations has a seemingly random nature. If the fact that the reference system is non-inertial and hence there exist additional variables in the form of inertial effects is ignored, then non-local correlation of seemingly independent observations would seem surprising. This example could visualize not only the interference of corpuscle particles, but also the non-local character of quantum correlations when considering the effects of entanglement.

## References

- [1] Newton I. *Philosophiae naturalis principia mathematica*. London, 1687. 220 p.
- [2] M. V. Ostrogradskii *Memoire sur les equations differentielles relatives aux problemes des isoperimetres* // *Memoires de l'Academie Imperiale des Sciences de Saint-Peterbourg*, v. 6, 1850. P. 385.
- [3] Lagrange J.I. *Mecanique analitique*. Paris, De Saint, 1788. 131 p.
- [4] Appel P., *Traite' de Me'caique Rationelle*, Paris, Ganthier-Villars e'diteur, 1953.
- [5] Bohm D. A suggested interpretation of the quantum theory in terms of "hidden" variables I // *Physical Review*. 1952. v. 85, P. 166.